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Master QFin, Mathematical Finance
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Hints

- This is a closed-book exam, one-page cheat sheet is permitted.
- Please use a new page for every assignment if possible.
- Good luck!

1. Itô calculus and Feynman Kac (9 points)

a) (3 points) Suppose that $dS_t = \mu S_t dt + \sigma S_t dW_t$. Compute the dynamics of $Y_t = \sqrt{S_t}$ and show that Y is again a geometric Brownian motion with parameters $\widetilde{\mu} = \frac{1}{2}\mu - \frac{1}{8}\sigma^2$ and $\widetilde{\sigma} = \frac{1}{2}\sigma dW_t$. Compute the quadratic variation $[Y]_t$. $\int_0^t Y_s^2 \cdot \sigma^2 \cdot \frac{1}{4}\, dt$

b) (2 points) Consider a process X that solves the SDE $dX_t = \gamma X_t dt + \kappa dW_t$, $X_0 = x \in \mathbb{R}$, $\gamma, \kappa > 0$. Use Itô's product formula or the time dependent Itô formula to obtain an SDE for the process $Y_t = e^{-\gamma t} X_t$. Conclude that $Y_t = x + \kappa \int_0^t e^{-\gamma s} dW_s$. Y_t given $\to$ check

c) (4 points) Use the Feynman Kac formula to solve the following terminal value problem. $f_t(t,x) + \mu x f_x + \frac{1}{2}\sigma^2 x^2 f_{xx} = 0$, $(t,x) \in [0,T) \times \mathbb{R}^+$, with terminal condition $f(T,x) = 1_{\{x \le K\}}$ for some $K > 0$ ($\mu \in \mathbb{R}$ and $\sigma > 0$ are given constants) $N\left(\frac{\ln(\frac{K}{x}) - (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)$

2. Black Scholes model and volatility estimation (4 points)

a) (2 points) Empirical properties of Black Scholes:
Describe two advantages and two disadvantages of the Black Scholes model.

b) (2 points) Volatility Estimation:
Describe historical and implied volatility.

3. Black Scholes model and logarithmic stock price.

(7 points) Consider in the context of the Black Scholes model with drift μ, volatility σ and short rate r a terminal value claim with payoff $h(S_T) = \ln S_T$.

a) (4 points) Use the risk-neutral pricing formula to compute the price of this option.
$e^{-r(T-t)}(\ln(S) + (r - \frac{1}{2}\sigma^2)(T-t))$

b) (2 points) Give a selffinancing replicating strategy (stock and money market account position) for the claim. $\Delta = e^{-r(T-t)} \cdot \frac{1}{S}$, $MM = e^{-rT}(\ln(S) + (r - \frac{1}{2}\sigma^2)(T-t) - 1)$

c) (1 point) Compute the Gamma of the terminal value claim.

$\Gamma = -e^{-r(T-t)} S^{-2}$